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HEAVY QUARKS IN CHARGED–CURRENT DEEP INELASTIC SCATTERING : A SYSTEMATIC PHENOMENOLOGICAL STUDY

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Abstract

We present a systematic QCD analysis of the strange–charm and bottom–top contributions to transverse and longitudinal structure functions in charged–current deep inelastic scattering. Various $\mathcal{O}(\alpha_s^1)$ schemes are studied and compared. The dependence of their predictions on the factorization scale μ^2 is investigated. The theoretical uncertainties resulting from the choice of the scheme and of μ^2 are estimated.

Neutrino Deep Inelastic Scattering (νDIS) is an invaluable source of information on nucleon parton densities. In particular, neutrino reactions play a fundamental rôle in the determination of the strange quark distribution [1].

Both to theorists and to experimentalists neutrino–induced deep inelastic processes pose a number of subtle problems, due to the peculiar structure of the weak currents. These currents are not conserved, hence large longitudinal contributions are expected to exist [2], which badly break the Callan–Gross relation. Moreover, in charged–current weak DIS, flavours of different masses are mixed and interesting threshold effects occur [3]. For instance, the excitation of strangeness (a light flavour) produces charm (a heavy flavour) via the lowest order transition $W^+ s \to c$, and there is the simultaneous production of $\bar{s}$ and c in the higher–order W–gluon fusion reaction $W^+ g \to \bar{s}c$. A similar situation takes place in the bottom–top sector.

Two important questions arise in charged–current DIS. They possess a great theoretical significance and, at the same time, are constantly faced by experimentalists in their analyses.

The first question is whether a leading order (LO) [1] approach, based on the quark excitation processes $W^+ q' \to q,\ W^+ \bar{q} \to \bar{q}'$ $(q' = s, b;\ q = c, t)$, and on the slow rescaling mechanism (see below), is sufficient to carry out a reliable data analysis. The slow rescaling (SR) procedure is appealing because it is relatively easy to perform, but it is known to give unreliable results, in some instance. Previous theoretical [1, 2, 3, 4, 5] and experimental [6, 7] studies on the determination of the strange density from neutrino DIS have indeed taught us the importance of quark-mass corrections and current non-conservation effects, which manifest themselves through the next–to–leading order (NLO) W–gluon fusion diagrams. However, when heavy quarks are involved, there is no unique way to take higher order effects into account in a partonic language. Various approaches exist, which hopefully (but not necessarily) are equivalent to each other.

Here comes the second problem announced above: the treatment of heavy flavours. It seems natural to make the notion of "heavy" and "light" depend on the physical scale of the process (this has been emphasized in [9]). At $Q^2 \simeq 10 - 30$ GeV^2, which is the typical average scale of present neutrino experiments, s is light whereas c is undoubtedly heavy. On the other hand, at $Q^2 \gg 10^2$ GeV^2, where the bottom contribution becomes visible, s, c and even b can be safely considered light quarks and only t is really heavy. How should one treat "heavy" flavours in the large–Q^2 limit, where collinear singularities emerge ? And how should one deal with the coexistence of "light" and

[1] We shall refer to the $\mathcal{O}(\alpha_s^0)$ quark excitation processes as "leading order", and to the $\mathcal{O}(\alpha_s^1)$ W-gluon fusion processes as "next-to-leading order". This terminology is admittedly ambiguous and is used here only for practical purposes and to adhere to some experimental papers [6, 7]; it differs from that adopted by other authors, *e.g.* [8].

"heavy" quarks in the same QCD processes, which is typical of charged–current DIS and represents a further source of ambiguity ?

Various schemes for a NLO QCD analysis of heavy quark contributions to electroweak structure functions are available on the market, but a detailed comparison of their predictions and their stability against changes of the factorization scale is still lacking. In this paper we want to bridge this gap, bringing to completion a program initiated in [1, 10][2]. We shall offer a panoramic and detailed view of the heavy flavour sectors of F_2 and F_L, with their dependence on x, Q^2, and in particular on the NLO scheme and the factorization scale. Results for F_3 have been presented elsewhere [10].

Let us start from the QCD factorization formula for the $q\bar{q}'$ contribution to the charged–current nucleon structure functions F_2 and F_L ($q = c, t$; $\bar{q}' = \bar{s}, \bar{b}$), that can be formally written as ($\otimes$ means convolution)

$$F_{2,L}^{qq'}(Q^2) = \sum_a f_a(\mu^2) \otimes \hat{F}_{2,L}^a(\mu^2, Q^2)\,. \tag{1}$$

Here the sum is made over all parton species, $\hat{F}_{2,L}^a$ are the structure functions for W scattering on parton a, and μ^2 is the factorization scale.

At order α_s^0 only quarks and antiquarks contribute to the sum in (1) and $\hat{F}_{2,L}^a$ are proportional to the cross sections for the excitations $W^+q' \to q$, $W^+\bar{q} \to \bar{q}'$, which are essentially delta functions of the slow rescaling variables [12]. For cs one has, *e.g.*

$$F_2^{cs}(x) = 2\,\xi\,[s(\xi) + \bar{c}(x)]\,, \tag{2}$$

where $\xi = (1 + m_c^2/Q^2)\,x$. The longitudinal structure function, although nonvanishing, is suppressed by a factor m_c^2/Q^2 (m_c is the mass of the charmed quark). Notice that eq. (2) is a parton model formula: the only Q^2 dependence is of a kinematical nature.

The slow rescaling (SR) procedure consists in using eq. (2) with a Q^2 dependence *à la* Altarelli–Parisi of the parton distributions. For cs

$$\mathrm{SR}: \quad F_2^{cs}(x, Q^2) = 2\,\xi\,[s(\xi, Q^2) + \bar{c}(x, Q^2)]\,. \tag{3}$$

We recall that in some experimental papers [6, 7] a "leading order analysis" of data is performed, which is in fact the slow rescaling analysis sketched above.

At order α_s the main contribution to the qq' component of the structure functions is given by the W–gluon fusion (GF) term, which reads

$$\mathrm{GF}: \quad F_{2,L}^{qq'}(x, Q^2) = \frac{\alpha_s(\mu^2)}{2\pi} \int_{ax}^{1} \frac{\mathrm{d}y}{y}\, y\, g(y, \mu^2)\, \hat{F}_{2,L}^g\left(\frac{x}{y}, Q^2\right)\,, \tag{4}$$

[2] A preprint has recently appeared [11] which partly overlaps the study carried out in Ref. [10] and here. However a different perspective is adopted in [11].

where $a = 1 + (m^2 + m'^2)/Q^2$ and $g(y, \mu^2)$ is the gluon density at the factorization scale. The explicit expressions for $\hat{F}_2^g$ and $\hat{F}_L^g$ are, in the limit $m' \to 0$ [13, 8][3]

$$\hat{F}_2^g(z,Q^2) = 2z \left\{ (1 - \frac{m^2}{\hat{s}}) \left[8z(1-z) - 1 - \frac{m^2}{Q^2}(1 - \frac{m^2}{Q^2})\, z(1-z) \right] + (1 + \frac{m^2}{Q^2})\, \mathcal{P}_{qg}(\tilde{z}) \left[\log\frac{\hat{s}}{m^2} + \log\frac{(\hat{s}-m^2)^2}{\hat{s}\, m'^2} \right] + 6z(1-2z)\frac{m^2}{Q^2}\log\frac{\hat{s}}{m^2} \right\} \quad (5)$$

$$\hat{F}_L^g(z,Q^2) = 2z \left\{ (1 - \frac{m^2}{\hat{s}}) \left[4z(1-z) - \frac{m^2}{Q^2}(1 - \frac{m^2}{Q^2})\, 2z(1-z) \right] + \frac{m^2}{Q^2}\, \mathcal{P}_{qg}(\tilde{z}) \left[\log\frac{\hat{s}}{m^2} + \log\frac{(\hat{s}-m^2)^2}{\hat{s}\, m'^2} \right] + 4z(1-2z)\frac{m^2}{Q^2}\log\frac{\hat{s}}{m^2} \right\}, \quad (6)$$

In eqs. (5,6) $\hat{s} = Q^2(1-z)/z$ and $\mathcal{P}_{qg}$ is the usual $g \to q\bar{q}$ splitting function expressed in terms of the rescaled variable $\tilde{z} = z(1 + m^2/Q^2)$.

When the quark q' is a light flavour, which means either $m' = 0$ or $Q^2 \gg m'^2$, the logarithm $L(m', m) = \log[(\hat{s} - m^2)^2/\hat{s}\, m'^2]$ develops a (collinear) singularity. Same considerations apply also to the logarithm $L(m, m') = \log(\hat{s}/m^2)$, when $Q^2 \gg m^2$. There are various methods to subtract (or to deal with) such collinear divergences.

Let us focus for the moment on the charm–strange sector. We shall discuss three different prescriptions. Two of them are variations of the so–called "fixed flavour scheme" (FFS) [8]. In this scheme the charmed quark is not considered to be a parton, *i.e.* an internal constituent of the nucleon, and its $\mathcal{O}(\alpha_s^0)$ excitation diagram is omitted. We call FFS(a) and FFS(b) the two realizations of the fixed flavour scheme that we shall consider in the following. They differ for the treatment of the strange quark: in FFS(a) strangeness is considered as a heavy flavour, with a mass $m_s \simeq 0.3 - 0.5$ GeV^2; in FFS(b) the strange quark is viewed as a massless parton. The third prescription is the "variable flavour scheme" (VFS) [9] in which charm is considered as a partonic constituent of the nucleon at large Q^2 and μ^2 ($Q^2, \mu^2 \gg m_c^2$) and the $\mathcal{O}(\alpha_s^0)$ charm excitation diagram is taken into account, with a proper subtraction term.

In the FFS(a) scheme one treats the strange quark (but not the charmed one) as a parton, setting m' to zero and subtracting in eqs. (5,6) the terms proportional to $\log Q^2/m'^2$, which are embodied in the Altarelli–Parisi evolution of the strange distribution. In explicit form, the FFS(a) prescription reads

$$\text{FFS(a)}: \quad F_{2,L}^{cs}(x,Q^2) = 2\,\xi\, s(\xi,Q^2) + \frac{\alpha_s(\mu^2)}{2\pi}\int_{ax}^{1}\frac{\mathrm{d}y}{y}\, y\, g(y,\mu^2)\, \tilde{F}_{2,L}^g\left(\frac{x}{y}, Q^2\right), \quad (7)$$

[3]For the sake of simplicity, we shall assume in our formulas that both the physical scale Q^2 and the squared mass m^2 of the quark q are much larger than the squared mass m'^2 of the quark q'; this is of course always true for the cs sector, and is true also for the bt sector at $Q^2 \gg 10^2\text{GeV}^2$. In the calculations the appropriate masses are used and no approximation is made.

where $\tilde{F}^g_{2,L}$ means $\hat{F}^g_{2,L}$, eqs. (5,6), with the terms proportional to $\log Q^2/m'^2$ subtracted off.

In the FFS(b) scheme eq. (4) is the only contribution to $F^{cs}_{2,L}$. A small but finite mass for the strange quark is retained and the full dependence on m and m' in the gluonic structure functions (5) and (6). Of course no singularity appears. Explicitly one has

$$\mathrm{FFS(b)}: \quad F^{cs}_{2,L}(x, Q^2) = \frac{\alpha_s(\mu^2)}{2\pi} \int_{ax}^{1} \frac{\mathrm{d}y}{y}\, y\, g(y, \mu^2)\, \hat{F}^g_{2,L}\left(\frac{x}{y}, Q^2\right), \tag{8}$$

with $m \equiv m_c \neq 0$ and $m' \equiv m_s \neq 0$.

In the fixed flavour schemes the number n_f of active flavours is set to 3, irrespective of the scale of α_s.

In the VFS scheme the strange quark is a massless parton and the charmed quark too is treated as a parton at large Q^2. Both the strange and the charm $\mathcal{O}(\alpha_s^0)$ excitation diagrams are considered but the Q^2 dependence of the parton distributions is kept in the kernel of the $\mathcal{O}(\alpha_s^1)$ factorization integral. The collinear singularities are subtracted out by setting

$$L(m, m') \to \log\frac{\hat{s}}{\mu^2}, \quad L(m', m) \to \log\frac{(\hat{s} - m^2)^2}{\hat{s}\,\mu^2}, \tag{9}$$

where the subtraction scale is customarily taken to be equal to the factorization scale. Thus the VFS prescription is

$$\begin{aligned} \mathrm{VFS}: \quad F^{cs}_{2,L}(x, Q^2) &= 2\,\xi\,[s(\xi, \mu^2) + c(x, \mu^2)] \\ &+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{ax}^{1} \frac{\mathrm{d}y}{y}\, y\, g(y, \mu^2)\, \bar{F}^g_{2,L}\left(\frac{x}{y}, Q^2\right), \end{aligned} \tag{10}$$

where $\bar{F}^g_{2,L}$ means the gluonic structure functions, eqs. (5,6), with the two subtractions (9). Notice also that in this scheme the number of active flavours is 4.

The bottom–top sector is treated along similar lines, but, out of the three approaches that we have just outlined, only FFS(b) and VFS survive.

What experimentalists do in their analyses is to use one of the methods presented above (including the slow rescaling prescription), choose a factorization scale and extract the charm and strange (or bottom and top) densities from the measured structure functions. It is clear that there at least two sources of uncertainty in this procedure: one coming from the choice of the scheme, the other coming from the choice of μ^2. The strategy adopted here reverses the experimental procedure: we use a set of parton distributions provided by a global fit and study the dependence on Q^2, μ^2 and on the scheme of the charged–current structure functions in the heavy quark sector.

In our calculations we use the MRS-A fit of parton densities [14]. For consistency we adopt the same set of quark masses of Ref. [14], that is $m_c^2 = 2.7$ GeV^2, $m_b^2 = 30$ GeV^2. We use also $m_t = 180$ GeV and, for the FFS(b), $m_s = 0.3$ GeV. As for the factorization scale, authors who adhere to different NLO schemes give different suggestions: in the fixed flavour scheme of Ref. [15], a value $\mu^2 = m_c^2$ is preferred, whereas in the variable flavour scheme of Ref. [9] a value which asymptotically becomes of order of Q^2 is suggested, and $\mu^2 = Q^2/2$ is practically used. For homogeneity we decided to choose the same value for all schemes, thus we take $\mu^2 = Q^2/2$ for the calculations presented in Figs. 1-4 and postpone the illustration of the μ^2 dependence to Fig. 5.

In Fig. 1 we present F_2^{cs} at two different values ($Q^2 = 25, 100$ GeV^2). The results of all prescriptions [FFS(a), FFS(b), VFS, SR] are shown. Consider, for instance, F_2^{cs} at $Q^2 = 25$ GeV^2. If we take the difference between the results of the various prescriptions as a measure of the theoretical uncertainty arising from the choice of the scheme, this uncertainty amounts to $\pm 3\%$ at $x = 10^{-3}$, and to $\pm 20\%$ at $x = 10^{-1}$. We have checked that the three NLO schemes give largely different predictions at lower x ($x < 10^{-4}$); however a correct analysis of this region requires the use of the k_t factorization method, which is beyond the purposes of this work. A forthcoming paper is specifically devoted to this issue [16]. Note that the slow rescaling curve almost coincides with the VFS result. On the other hand, the largest discrepancy occurs between the FFS(a) and the FFS(b) predictions, due to the $\mathcal{O}(\alpha_s^0)$ term which is absent in FFS(b). We mention that the FFS(b) result for cs is affected by an additional (and large) uncertainty coming from the choice of m_s. This makes FFS(b) intrinsically unreliable.

If we confine ourselves to VFS and FFS(a), the theoretical uncertainty is $\pm(3-7)\%$ over the whole x range at $Q^2 = 25$ GeV^2 and gets smaller with increasing Q^2, being $\pm(2-4)\%$ at $Q^2 = 100$ GeV^2. Thus it will be important to have data on F_2^{cs} at larger Q^2.

In Fig. 2 the curves for the longitudinal charm–strange structure function F_L^{cs} are displayed. The FFS(a) and FFS(b) results are much closer to each other in this case than in the F_2^{cs} case. The theoretical uncertainty amounts now to $\pm 10\%$ at $Q^2 = 25$ GeV^2 over the whole x range. Obviously the SR curve for F_L^{cs} is much lower, being suppressed by a factor m_c^2/Q^2. The slow rescaling approach is expected to be unreliable when the longitudinal structure function dominates. The relevance of the longitudinal contribution is quantified by the ratio $\rho \equiv F_L/F_2$, which is a measure of the violation of the Callan–Gross relation. In the cs sector we find, in the FFS(a) scheme, $\rho^{cs} \simeq 0.30$ at $x = 10^{-3} \div 10^{-2}$.

The breaking of the Callan–Gross relation is stronger in the bt sector. In Fig. 3 we show the SR, FFS(b) and VFS predictions for F_2^{bt} at two Q^2 values ($Q^2 = 10^3, 10^4$ GeV^2). The difference between FFS and VFS amounts to $\sim 10\%$, whereas the discrep-

ancy between slow rescaling and NLO schemes is more dramatic (up to $\sim 70\%$). We do not plot F_L^{bt}, limiting ourselves to mention that it dominates F_2^{bt}. We find in fact $\rho^{bt}(x=10^{-3}) = 0.97$ (*i.e.* $F_L^{bt} \simeq F_2^{bt}$) at $Q^2 = 10^3$ GeV^2, and $\rho^{bt}(x=10^{-3}) = 0.79$ at $Q^2 = 10^4$ GeV^2 (these values are almost scheme independent).

For completeness we show in Fig. 4 the Q^2 dependence of the structure functions F_2^{cs} and F_L^{cs} in two schemes, VFS and FFS(a). The trend is similar in the two cases.

Now, is there a way to prefer a scheme to another ? A practical criterion is to look at the dependence of the results on the factorization scale μ^2. In Fig. 5 we present the μ^2 dependence of F_2^{cs} at $Q^2 = 25$ GeV^2 in each scheme. It can be easily seen that VFS and FFS(a) are more stable against changes in μ^2. A slight preference should be attributed to the latter scheme, which gives a practically constant prediction in the range $\mu^2 = (5 - 100)$ GeV^2, whereas in the same range the VFS result changes by $\sim 25\%$ at $x = 10^{-3}$, and by $\sim 15\%$ at $x = 0.1$. We have checked that also at $Q^2 = 100$ GeV^2 FFS(a) is more stable than VFS in the range $\mu^2 = (5 - 1000)$ GeV^2. The main variation of VFS occurs for small μ^2 values, so that if VFS is used a $\mu^2 \gtrsim Q^2/2$ is preferable.

We remark here that in the next–to–leading CCFR analysis of dimuon data [7], which resorts to the VFS scheme, the factorization scale is chosen to be equal to twice the maximum available transverse momentum of the final charm quark, $\mu = 2p_\perp^{max}$. At $x = 10^{-2}$ this scale is about two orders of magnitude larger than the CCFR average Q^2, which is 22 GeV^2. The quoted uncertainty on the strange distribution due to the arbitrariness of the choice of μ^2 is about 5% and is obtained by varying μ between $p_\perp^{max}$ and $3p_\perp^{max}$. Our results show that the CCFR Collaboration underestimates the theoretical uncertainties of its determination.

Let us summarize our work. In this paper we provided an (x, Q^2, μ^2) map of the heavy quark contributions to charged current structure functions in some customary QCD schemes. The best way to choose the optimal scheme and factorization scale is to study the perturbative stability, which is possible only by pushing at least to order α_s^2 the knowledge of the factorization kernels. Some partial results heve been obtained in the electromagnetic case [15, 17], but a similar treatment of the weak DIS is still lacking and will be object of a future investigation. However some conclusions can be already drawn. While the slow rescaling mechanism should be definitely abandoned – at least when longitudinal contributions are present (it may nevertheless be useful for the analysis of purely transverse structure functions such as xF_3 [10]) –, it seems that, in the charm-strange sector, the fixed flavour scheme with subtraction for the strange quark – the scheme we have called FFS(a) – is preferable due to its greater stability. In any case, the uncertainties coming from the choice of the scheme and of the factorization scale add up to some tens per cent, and require a better consideration

both on the experimental and on the phenomenological side. More data, especially at larger Q^2, are needed in order to achieve a well–established knowledge of the strange and charm distributions.

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Figure Captions

Fig. 1 F_2^{cs} at $Q^2 = 25,\ 100$ GeV^2. The solid line corresponds to the VFS result [see the text], the dot-dashed one to FFS(a), the dashed one to FFS(b), the dotted one to SR.

Fig.2 F_L^{cs} at $Q^2 = 25,\ 100$ GeV^2. The curves are labelled as in Fig. 1.

Fig.3 F_2^{bt} at $Q^2 = 10^3,\ 10^4$ GeV^2. The curves are labelled as in Fig. 1.

Fig.4 The Q^2 dependence of F_2^{cs} and F_L^{cs}. Only the VFS prediction (solid curve) and the FFS(a) (dot-dashed) prediction are shown.

Fig.5 The dependence of F_2^{cs} on the factorization scale μ^2 at $x = 0.001,\ 0.01$ and 0.1 and $Q^2 = 25$ GeV^2. The curves are labelled as in Fig. 1.